PGR MATHEMATICS CONFERENCE

Wednesday 29th March 2023

Time	Event	Room
09:20 - 09:30	Welcome	39AA04
09:30 - 10:30	Guest Speaker Josephine Solowiej-Wedderburn	39AA04
10:30-11:00	Coffee Break	39AA04
11:00 - 12:00	Speaker 1 Lewis Napper	39AA04
12:00 - 13:00	Speaker 2 Rachel Bernasconi	39AA04
13:00-15:00	Lunch	Please join us in Hillside
15:00 - 16:00	Speaker 3 Dominic Stone	24AA04
16:00 - 16:30	Coffee and Posters	39AA04
16:30 - 17:30	Speaker 4 Dan Catlin	24AA04
17:30 - 17:45	Finish	24AA04

Guest Speaker: Josephine Solowiej-Wedderburn

How does the environment shape bacterial communities?

Bacteria live in diverse communities, forming complex networks of interacting species. A central question in bacterial ecology is why some species engage in cooperative interactions, whereas others compete? But this question often neglects the role of the environment. Here, we take a theoretical, data-driven approach to address this gap. We use genome-scale metabolic networks from two different open-access platforms (AGORA and CarveMe) to assess pairwise interactions of different microbes in varying environmental conditions (provision of different environmental compounds). This big data approach allows us to determine commonalities of environments that could facilitate the potential for cooperation or competition between a pair of species. By scanning thousands of environments, we are also able to identify shifts in resource availability that can cause a transition in the interaction between a pair of microbes, e.g. from competition to cooperation. Finally, our approach allows us to develop the null models necessary to test hypotheses on which interaction types should be expected in given environments.

Speaker 1: Lewis Napper

Monge–Ampère Geometry and the Navier–Stokes Equations

For partial differential equations of Monge–Ampère type, it has been shown [1, 2] that solutions correspond to Lagrangian submanifolds of the associated phase space. Build- ing from the observation that the Poisson equation for the pressure of an incompressible, two-dimensional, Navier–Stokes flow is a Monge–Ampère equation, this talk introduces a framework for studying fluid dynamics using properties of the aforementioned submani- folds. In particular, it is noted that such a submanifold may be equipped with a metric whose signature acts as a diagnostic for the dominance of vorticity and strain. We provide an illustrative example in two dimensions and probe the motivational question [3,4]: 'What is a vortex?' We conclude with comments on extensions to fluid flows in higher dimensions and some open questions.

References

[1] V.V.Lychagin,V.N.Rubtsov,andI.V.Chekalov,AclassificationofMonge–Ampèreequations, Ann. Sci. Ec. Norm. Sup. 26 (1993) 281.

[2] A. Kushner, V. Lychagin, and V. Rubtsov, Contact geometry and non-linear differential equa- tions, Cambridge University Press, 2007.

[3] M. Larchevêque, Pressure field, vorticity field, and coherent structures in two-dimensional in- compressible turbulent flows, Theor. Comp. Fluid Dynamics 5 (1993) 215.

[4] J. D. Gibbon, A. S. Fokas, and C. R. Doering, Dynamically stretched vortices as solutions of the 3D Navier–Stokes equations, Phys. D. Nonlin. Phen. 132 (1999) 497.

Speaker 2: Rachel Bernasconi

Owls, larks and tongues – sleep timing in the two-process model of sleep-wake regulation

The average person is estimated to spend a third of their life asleep. Despite this, a large proportion of the population regularly experience insufficient sleep or suffer from sleep disorders. These have been linked to poorer health outcomes, with a multinational survey by RAND suggesting inadequate sleep increases mortality by up to 13%. The complexity and importance of sleep means that interdisciplinary approaches including mathematical modelling are valuable to develop insight. Since its formulation in the early 1980s, the two-process model of sleep-wake regulation has been at the forefront of sleep modelling. The model suggests that two oscillators – one capturing the daily cycle of the body's 'clock' system and the other representing sleep need – interact to generate sleep/wake rhythms. The two-process model has provided many insights into both the overall patterns of sleep and how performance is affected when we are sleep deprived. However, there remain aspects of sleep that have not yet been described using the two-process framework. For example, preferred sleep timing varies across the population, from 'early birds' (larks) to 'night owls'. While a full explanation of these preferences requires the addition of another oscillator describing the interaction of the sleep-wake regulatory system with light, we will show that the original two-process model can replicate the experimental observation that larks wake up at a later time in their biological clock than owls. Specifically, we derive expressions for the relative phase of the two oscillators in the two-process model and relate them to the underpinning Arnold 'tongue' structure that captures when sleep and the body clock are synchronised. Our work is of biological significance as it not only allows us to reproduce variations in sleep phase across healthy individuals like larks and owls, but also provides the groundwork for study into various disease states where poorly timed sleep is a contributing factor.

Speaker 3: Dominic Stone

Maximal Regularity of Nonstationary Stokes problem

The Nonstationary Stokes problem is related to two similar equations namely the stationary Stokes problem and the heat equation. It is well known that both these equations enjoy maximal regularity for certain Lebesgue spaces.

However, the case of nonstationary Stokes is less well studied. It has been shown (see [1]) that in the nonstationary case additional requirements are needed on the temporal regularity of the divergence term to guarantee maximal regularity.

This presentation constructs a counterexample function f(x,t) that fails to produce maximal regularity of the nonstationary Stokes problem using partial Fourier transforms.

References

[1] N. Filinov and T. Shilkin. "On the Stokes problem with nonzero diver- gence". In: Journal of Mathematical Sciences 166.1 (Mar. 2010), pp. 106– 117.

Speaker 4: Dan Catlin

Analysing The Uncertainty of Wiretaps in Criminal Networks

Criminal investigations involving wiretaps are often long, expensive and conducted with very little information of the entire criminal organisation being investigated. Once a few members of the organisation are identified, surveillance can begin, and further co-conspirators identified with the aim to find the key players in the network. But how do we know who are the important persons in a network which is usually very decentralised, and how can we be sure of our findings to progress the investigation?

We look at the communications data of criminal organisation as a time-dependant network of symmetric graphs and use the time evolving Katz network centrality to determine the most important nodes in the network based on the differing amount of information flow in the network and their broadcasting or receiving of it.

From here we modify the network by introducing Bernoulli variables to encapsulate conversations that could exist but were not heard by law enforcement, using this we can create new node rankings of importance based on the likelihood of missing conversations for individuals under increased surveillance.

From this we can then explicitly calculate the expectation and variance of the enhanced network matrices and study the stability of its values under small perturbations. We conduct this investigation on synthetically generated network data with differing characteristics and present results using a real-life case where data was collected over a period of 4 years.

By analysing the stability of these systems, we can improve the efficacy and efficiency of these investigations to allow law enforcement to better allocate resources and come to more favourable conclusions.

Poster Presentations

Kieran Boniface
Mechanotransduction in organoid development

Jessica Furber
Spending Energy for a Reward: Understanding the Fine Movements of Badgers

Ryan Poole
The Effects of Compliance on the stability of Jets and Wakes

Elliott Sullinge-Farrall
Optimisation of a 1D Family of Polyconvex Functionals

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Organisers: Jessica Furber, Ryan Poole, Elliott Sullinge-Farrall, and Vaibhav Gautam